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The Effects of Spatial Correlations on Merger Trees of Dark Matter Haloes

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ABSTRACT
The effects of spatial correlations of density fluctuations on merger histories of dark matter haloes (so-called '*merger trees*') are analysed. We compare the mass functions of dark haloes derived by a new method for calculating merger trees, that proposed by Rodrigues & Thomas (RT), with those given by other methods such as the Block model, the Press-Schechter formula and our own formula in which the mass functions are analytically expressed in a way that takes into consideration the spatial correlations. It is found that the mass functions given by the new method are well fit by those given by our formula. We believe that new method (RT) *naturally* and correctly takes into account the spatial correlations of the density fluctuations due to a calculated, grid-based realisation of the density fluctuations and so is very useful for estimating the merger tree accurately in a way that takes into consideration spatial correlations.

Moreover, by applying our formula, we present an analytic expression which reproduces the mass function derived by the Block model. We therefore show clearly why and how the mass functions given by the new method and the Block model are different from each other. Furthermore, we note that the construction of merger trees is sensitive to the criterion of collapse and merging of overlapped haloes in cases in which two or more haloes happen to overlap. In fact, it is shown that the mass function is very much affected when the criterion of overlapping is changed.

Key words: galaxies: formation – galaxies: mass function – large-scale structure of the universe

1 INTRODUCTION

Recent observations by the HST and the Keck telescope have rapidly increased the available numbers of observations of faint, high redshift galaxies, providing us with significant new information about the birth and evolution of galaxies. In order to understand the significance of these observations, it is very important to theoretically understand the physical processes underlying galaxy formation.

In our standard understanding, it is considered that dark matter dominates in our Universe and that galaxies and clusters of galaxies have formed by the gravitational growth of initial small density fluctuations. The fluctuations of dark matter have collapsed and virialised by self-gravitational instability into objects which are called 'dark matter haloes' or 'dark haloes'. The larger haloes are generally considered to have formed hierarchically by clustering of smaller haloes (it is so called 'hierarchical clustering').

The baryonic gas also has collapsed and virialised following the collapse of the dark matter. Furthermore, in the process of galaxy formation, baryonic gas must dissipate the internal energy by radiative cooling and shrink because galaxies are much denser systems than the virialised dark haloes and stars must be formed inside the cold and dense gaseous systems. In analysing the formation and evolution of the galaxies, we must treat various physical processes over a large dynamic range from $\sim 1-10^2 M_\odot$ (star formation, heating processes of gases from supernovae, dynamical and chemical evolution of gases, etc.) through $\sim 10^{6-12} M_\odot$ (mergers of galaxies, tidal interactions, etc.) to $\sim 10^{13-16} M_\odot$ (clusters of galaxies, large scale structure of the Universe, etc.). So it is difficult to attack the problem of formation and evolution of galaxies in a way that connects all of the above complicated physical processes.

One method of analysing galaxy formation is numerical simulations (e.g., Navarro & White 1993, Katz & Gunn

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1991, Katz 1992, Steinmetz & Muller 1994, Evrard, Summers & Davis 1994) which directly pursue the gravitational growth of dark matter and thermal processes of gaseous systems. The advantage of simulation is that we can trace the complicated processes of the systems quantitatively. However, it is impossible to deal with the very wide range of the mass scales simultaneously in the current limited ability of computers. Furthermore, since the CPU time is heavily consumed in the simulations, it is difficult to analyse galaxy formation statistically by pursuing many samples of the system with different initial conditions and parameters in the physical processes. So we can say that the problem of galaxy formation has never been resolved completely by numerical simulations without any uncertainty.

On the other hand, there is another approach to solving galaxy formation, that is, the semi-analytical approach, which has been developing. Some works in this approach are briefly reviewed below. The pioneer works in this approach are, for example, Rees & Ostriker (1977) and White & Rees (1978). Rees & Ostriker remarked upon the importance of the dissipational process of baryonic gases through radiative cooling. They asserted that a dense object like a galaxy must have a cooling time scale τ_{cool} shorter than the dynamical time scale τ_{dyn} of the system at the virialised stage because the object must dissipate the energy effectively in order to become a more dense system and make stars in the cold and dense gases. Therefore, one of the criteria for a virialised object to become a galaxy in the future is $\tau_{cool} < \tau_{dyn}$ at the virialised stage. Then Rees & Ostriker showed in the cooling diagram that the range $\tau_{cool} < \tau_{dyn}$ corresponds to the characteristic mass range of galaxies, *i.e.*, $10^8 M_\odot \sim 10^{12} M_\odot$.

White & Rees (1978) showed the luminosity function of galaxies by using the Press-Schechter formula (Press & Schechter 1974;hereafter PS, *see* below) in which the mass function of dark haloes is analytically estimated. Furthermore, White & Frenk (1991) extended this approach to consider the *merging process* of dark haloes approximately by using an extended PS formula (Bower 1991). In this formula they take into account the conditional probability of finding a region with mass M_1 at redshift z_1 which is also included in a region with mass M_2 at z_2. They calculate the present stellar and gas abundance by tracing the merger process of the dark haloes from an initial time till the present time in each mass scale of the halo on the assumption that there is a relation between the halo mass, the gas cooling rate, the accretion rate of gas onto the halo and the star formation rate. The extension of the PS formula is also discussed by Lacey & Cole (1993). By using the peak formula (Peacock & Heavens 1985, Bardeen et al. 1986; hereafter BBKS), Lacey & Silk (1991, 1993) also investigate the time scale of the collapse of galaxy groups after the galaxies have collapsed on the assumption that the mass ratio of groups to galaxies is constant.

However, the status of the gaseous systems and formations of stars in each dark halo at an epoch depend on the thermal history of the gaseous systems, merging of the galaxies and the merging history of the halo. So it is very important to know the merging history of each dark halo at an epoch in order to evaluate the status of the gaseous systems and the formation rate of stars in the halo at that epoch. Kauffman & White (1993; hereafter KW) constructed a *merger tree* that expresses the merging history of the dark haloes by considering progenitors of each halo at every time by using a Monte Carlo method of the extended PS formula. Kauffman, White & Guiderdoni (1993) then calculated the statistical properties of galaxies in their model. Another approach to construction of a merger tree is the Block model, which was developed by Cole & Kaiser (1988). The Block model takes the Monte Carlo procedure described as follows: First of all, the density contrast is assigned to the largest block with mass $M_0(\sim 10^{16} M_\odot)$ which has the variance of the density contrast, $\sigma^2(M_0)$. Next, the largest block is divided into two blocks with the same mass $M_1(= M_0/2)$. The additional positive density fluctuation generated by a random Gaussian distribution with variance $\sigma^2(M_1) - \sigma^2(M_0)$ is assigned to one of the two divided blocks with mass $M_1(= M_0/2)$ and a negative fluctuation with the same absolute value of the amplitude as the positive one is assigned to the other divided block. The same procedure is repeated for $M_1, M_2, \cdots$ down to the smallest mass scale under consideration, thereby constructing the merger tree. Cole et al. (1994) also calculated the statistical properties of formation and evolution of galaxies in their Block model.

Incidentally, it has been pointed out that the PS formula has the following crucial weak points. The PS formula is derived as follows: In an Einstein-de Sitter universe, spherical overdense regions collapse and virialise when their linear density contrast reaches $\delta_c = 1.69$ (see, e.g., Peebles 1993). Then, assuming that the density fluctuations obey a random Gaussian distribution and that the collapse of the haloes is spherically symmetric, we can get the volume fraction of the regions of collapsed objects whose masses are larger than the mass M, $f(\geq \delta_c, M)$. So, the region of the dark haloes with mass scale M is equal to $\partial f/\partial M$. Thus they proposed the formula for counting the number density of objects of mass scale M. However, only overdense regions were considered in the analysis. Even if the density contrast smoothed on the mass scale M is less than δ_c, there is the case such that the density contrast smoothed on the larger mass scale than M is greater than δ_c. We must consider this case to count exactly the number density of the dark haloes. This problem is called the 'cloud-in-cloud' problem. Press & Schechter (1974) *simply* multiplied the number density by a 'fudge factor' of 2 without a good reason.

Peacock & Heavens (1990) and Bond et al. (1991) attempted to solve the cloud-in-cloud problem by using a peak formula. They considered the *upcrossing* probability that a density contrast which is lower than δ_c with a smoothing scale M exceeds δ_c for the first time when increasing the smoothing scale. In the case that the density field is smoothed with the sharp k-space filter, they found that the factor of 2 introduced by PS is correct. The cloud-in-cloud problem is also solved for Poisson fluctuations by Epstein (1983).

In the cloud-in-cloud problem, however, we must consider the *spatial correlations of the density fluctuations* since objects have non-zero size in reality. Yano, Nagashima & Gouda (1996; hereafter YNG) analysed the cloud-in-cloud problem taking explicitly into account the spatial correlation of the density fields. They explicitly introduced the two-point correlation function in the mass function by using Jedamzik's formula (Jedamzik 1995) in which the number density of the collapsed objects is given in the form of the integral equation which is different from the PS formula.

It is also proved by YNG that the results derived from the Jedamzik formula are consistent with those derived from the PS formula on the assumption that the collapse is spherically symmetric when the spatial correlations are not taken into consideration. However, YNG showed that the spatial correlations greatly affect the mass function. Therefore, we believe that the merger tree is also affected by the spatial correlations whose effects have never explicitly been taken into account in the KW method and the Block model. So we believe it is very important to analyse the effects of spatial correlations of the density fields on the merger trees of dark haloes.

Recently, a new approach to construction of merger trees has been proposed by Rodrigues & Thomas (1995) (we call this the *Merging Cell model* for convenience throughout this paper). In their model, the random Gaussian density fluctuation field is realised on spatial grids as in constructing initial conditions of N-body simulations, so it is expected that this model naturally includes information about the spatial correlation. Then, by finding the region of the collapsed cells or blocks with each mass scale whose density contrast δ is δ_c, we can construct the merger tree (see §2.1). We expect that this model is more realistic and useful for galaxy formation although spherical collapse and random Gaussian density fluctuations are assumed.

In this paper, we show the mass functions of dark haloes by calculating the Merging Cell model. They are compared with the mass functions given by the PS formula and the Block model. We then explicitly show that the main origin of differences between the mass functions given by the different methods result from the effect of spatial correlations by comparing with the mass functions derived by YNG's formula (hereafter, the YNG formula). We find that the mass functions given by the YNG formula in taking explicitly into account the spatial correlations are consistent with those given by the Merging Cell model and so the Merging Cell model correctly includes the effects of the spatial correlations. We believe that this effect is very important for calculating the merger tree of the dark haloes. Furthermore, by applying the Jedamzik formula, we present an analytical expression of the mass function derived from the Block model and show quantitatively why and how the mass function given by the Block model is different from those derived from the PS formula and the Merging Cell model.

In the Merging Cell model, some haloes happen to overlap since the non-zero size of the haloes is considered explicitly. So, we must consider the criterion of collapse and merging for the overlapped haloes. We also show how the criterion affects the mass function in the Merging Cell model.

In §2, the Merging Cell model and the Block model are reviewed briefly. In §3, we give the analytical formulae for estimating the mass functions by using the Press-Schechter formula, the Jedamzik formula and the YNG formula. In §4, it is shown that the Merging Cell model is consistent with the mass function which is derived by the analytical formula in which the spatial correlations are taken into account. The mass function given by using the Jedamzik formalism which reproduces those given by the Block model is also presented. Furthermore, we also show how the mass function is changed if the overlapping criterion is changed. We devote §5 to conclusions and discussions.

2 MODELS OF MERGER TREES

In this section, we briefly review the Merging Cell model (hereafter MCM) and the Block model.

2.1 Merging Cell model

We briefly review the MCM according to the procedure and the notations shown in Rodrigues & Thomas (1995).

First, the random Gaussian density field is realised in a periodic cubical box of side L. In the random Gaussian distribution, Fourier mode of density contrast $\delta(=(\rho-\bar{\rho})/\bar{\rho}$, ρ is density and $\bar{\rho}$ is the mean density of the universe) obeys the following probability for its amplitude and phase(BBKS),

$$P(|\delta_{\mathbf{k}}|,\phi_{\mathbf{k}})d|\delta_{\mathbf{k}}|d\phi_{\mathbf{k}} = \frac{2|\delta_{\mathbf{k}}|}{P(k)}\exp\left\{-\frac{|\delta_{\mathbf{k}}|^2}{P(k)}\right\}d|\delta_{\mathbf{k}}|\frac{d\phi_{\mathbf{k}}}{2\pi}, \qquad (1)$$

where $\phi_{\mathbf{k}}$ is the random phase of $\delta_{\mathbf{k}}$, $\delta_{\mathbf{k}} = |\delta_{\mathbf{k}}|\exp(i\phi_{\mathbf{k}})$ and $P(k)$ is the power spectrum $\langle|\delta_{\mathbf{k}}|^2\rangle$, where the angle brackets mean the ensemble average of the universe. Then, the density contrast at each grid ('cell') is given by Fourier transform,

$$\delta(\mathbf{x}) = \frac{V}{(2\pi)^3}\int_0^{k_c}\delta_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}d^3k, \qquad (2)$$

where k_c is the cut-off wavenumber.

Next, averaging the density fluctuations within cubical *blocks* of side 2, 4, ..., L, the fluctuations of the various smoothing levels are constructed. At each smoothing level, displacing the smoothing grids by half a blocklength in each direction of each axis, eight sets of *overlapping grids* are constructed in order to reflect the position of density peaks approximately (see Fig.1).

Then, the density fluctuations within blocks and cells are combined into a single list and ordered in decreasing density. The fluctuations are investigated from the top of the list. It is decided by the following rules whether each block or cell can collapse. Note terminology that *halo* is a block or cell which has already collapsed, and an *investigating* region is a block or cell whose linear density contrast is just equal to δ_c at the reference time. We investigate whether or not an investigating region can collapse at that time according to the following rules.

(a) If an investigating region includes no haloes, the investigating region(*block* or *cell*) can collapse and can be identified as a new halo.

(b) When an investigating region includes a part of a halo, if the *overlapping region* has a larger mass than the minimum of the masses of the halo and the investigating region, then the investigating region can collapse (see Fig.2). This is the criterion of collapse of the investigating region and merging for the overlapped haloes. We call this criterion *the overlapping criterion* in this paper.

(c) In the case of (b), if the investigating region has two haloes whose overlapped region within the investigating region is more than a half of the region in each halo, the region cannot collapse. We set this condition in order to avoid long filamentary objects. This is the criterion for linking of haloes.

These criteria are those chosen by Rodrigues & Thomas. We will change the overlapping criterion (b) later and see

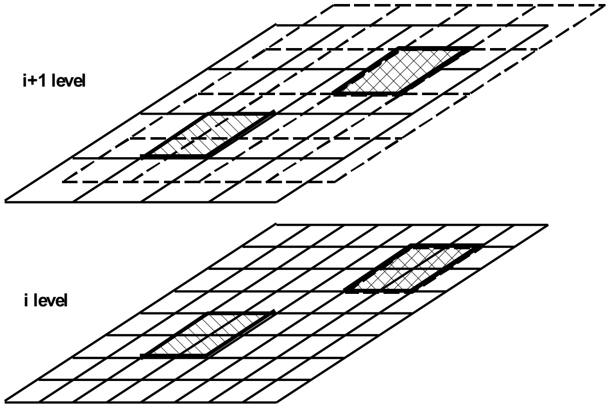

Figure 1. Scheme of averaging density fluctuations demonstrated in 2 dimensions (after Rodrigues & Thomas 1995). Lower grid represents i-th level. Upper grids represent $i+1$-th level with one of the overlapping grids displaced by half a blocklength. The hatched region in the lower grid is averaged to make a block at the $i+1$-th level and indicated as the region marked as a thick square in the upper grid. Blocks in the offset grid are averaged from the lower level in the same way.

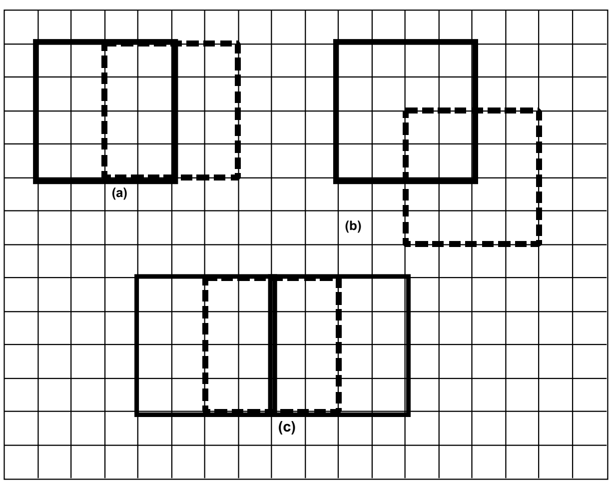

Figure 2. The overlapping criterion. The thick squares show a halo and the dashed squares show an investigating region. (a)The block can collapse and is merged into the halo because the overlapping region is larger than half of the lesser of the regions of the halo and the investigating region. (b)The block cannot collapse because the overlapping region is not larger than half of the lesser of the regions of the halo and the investigating region. (c)The linking condition. In the case shown here, the block cannot collapse because we would like to prevent the growth of the long filamentary structure (see text). Note that it is equivalent to consider either regions or masses for the overlapping criterion.

how the mass function is changed. The condition (c), the linking condition, prevents the growth of filamentary structures because galaxies that we observe are not filamentary. However, we should note that the filament structures of dark matter certainly appear as shown in numerical simulations. Then the criterion for linking is just an assumption.

It must be noted here that in the MCM, because overlapping grids are used, the mass spectrum is close to continuous, rather than restricted to powers of two as in the Block model.

2.2 Block model

In this subsection, the Block model is briefly reviewed according to Cole & Kaiser (1988). Now, we consider a large block with mass $M_0(\sim 10^{16}M_\odot)$, divide this into two, and realise blocks with various mass scales by successively dividing smaller and smaller blocks. We obtain a set of blocks of discrete masses, $M_i = M_0/2^i (i = 1, 2, \cdots, N)$ where N is an integer and N is 27 in our calculation as shown later.

Here $\sigma(M_i) = \sigma_i$ means the standard deviation of the density fluctuations smoothed on a mass scale M_i. When the power spectrum is written in the scale-free form $P(k) \propto k^n$, the standard deviation is given by

$$\begin{aligned}\sigma_i^2 &= \frac{4\pi V}{(2\pi)^3}\int_0^\infty W_i^2(k)P(k)k^2dk \\ &= \left(\frac{M_i}{M_*}\right)^{-\frac{3+n}{3}}, \qquad (3)\end{aligned}$$

where M_* is the mass on which the variance is unity, and $W_i(k)$ is the window function. We define the quantities Σ_i as follows:

$$\Sigma_0 = \sigma_0, \qquad (4)$$

$$\Sigma_i^2 = \sigma_i^2 - \sigma_{i-1}^2, \quad i \geq 1. \qquad (5)$$

We generate the random density fluctuations on each block as follows: First of all, a density contrast generated by the random Gaussian distribution with the variance Σ_0 is assigned to the largest block. This corresponds to the density contrast smoothed on the mass scale M_0. Then the positive random variable generated by the random Gaussian distribution with the variance Σ_1 is added to one of the two divided blocks with the mass $M_1 = M_0/2$ and a negative one with the same absolute value of the amplitude is added to the other divided block. Repeating this procedure from $i = 0$ until $i = N$, we obtain a tree of density contrasts of the blocks with various mass scales. The condition of collapse for a block at the redshift z is that the density contrast of the block equals $\delta_c = 1.69(1+z)$ (z; redshift). In this way, we can construct the merger tree of the dark haloes.

3 ANALYTIC APPROACH TO MASS FUNCTIONS

In this section, we present the PS formula, the Jedamzik formula and also the mass functions derived by our formula (YNG formula) which explicitly includes the effects of the spatial correlations.

3.1 Press-Schechter formula

The probability of finding the region whose linear density contrast smoothed on the mass scale M, δ_M, is greater than or equal to δ_c is assumed to be expressed by the random Gaussian distribution given by

$$f(\geq \delta_c, M) = \frac{1}{\sqrt{2\pi}\sigma(M)}\int_{\delta_c}^\infty \exp\left(-\frac{\delta^2}{2\sigma^2(M)}\right)d\delta. \qquad (6)$$

This probability corresponds to the ratio of the volume of the region above δ_c in the density contrast smoothed on the mass scale M to the total volume (in a fair sample of

the Universe). Therefore, the difference between $f(\geq \delta_c, M)$ and $f(\geq \delta_c, M + dM)$ represents the volume of the region for which $\delta_M = \delta_c$ precisely. The density contrast of an *isolated* collapsed object is precisely equal to δ_c because an object with $\delta > \delta_c$ would be eventually counted as an object of larger mass scale. The volume of each object with mass scale M is $M/\bar{\rho}$. Then we obtain the following relation,

$$\frac{Mn(M)}{\bar{\rho}}dM = -\frac{\partial f(\geq \delta_c, M)}{\partial M}dM, \qquad (7)$$

where $n(M)$ means the number density of objects with mass M, that is, the mass function. However, the underdense regions are not considered in the above equation. Hence, Press and Schechter *simply* multiply the number density by a factor of 2,

$$\frac{Mn(M)}{\bar{\rho}}dM = -2\frac{\partial f(> \delta_c, M)}{\partial M}dM. \qquad (8)$$

This factor of 2 has long been noted as a weak point in the PS formula (the so-called 'cloud-in-cloud' problem). Peacock & Heavens (1990) and Bond et al. (1991) proposed a solution to this problem by taking account of the probability P_{up} that subsequent filtering an larger scales might result in having $\delta > \delta_c$ at some point, even when at smaller filters, $\delta < \delta_c$ at the same point. They found that the factor of 2 in the PS formula could be correct only by using the sharp k-space filter given by

$$W_i(k) = \begin{cases} 1, & k \leq k_c(M_i) \\ 0, & k > k_c(M_i). \end{cases} \qquad (9)$$

3.2 Jedamzik formula

Jedamzik (1995) proposed another approach to the cloud-in-cloud problem.

Now, we consider the regions whose smoothed linear density contrasts on the mass scale M_1 are above δ_c. Each region must be included in an isolated collapsed object with mass $M_2 \geq M_1$. Therefore, we obtain the following equation,

$$f(\geq \delta_c, M_1) = \int_{M_1}^{\infty} P(M_1, M_2)\frac{M_2}{\bar{\rho}}n(M_2)dM_2, \qquad (10)$$

where $P(M_1, M_2)$ means the conditional probability of finding a region of mass scale M_1 in which δ_{M1} is greater than or equal to δ_c, provided it is included in an isolated overdense region of mass scale M_2. By 'the Jedamzik formula' we mean the procedure in which the mass functions are estimated by solving eq.(10). If $P(M_1, M_2)$ is given by the conditional probability $p(\delta_{M_1} \geq \delta_c | \delta_{M_2} = \delta_c)$, $P(M_1, M_2)$ is written as follows by using Bayes' Theorem,

$$\begin{aligned} P(M_1, M_2) &= p(\delta_{M_1} \geq \delta_c | \delta_{M_2} = \delta_c) \\ &= \frac{p(\delta_{M_1} \geq \delta_c, \delta_{M_2} = \delta_c)}{p(\delta_{M_2} = \delta_c)} \\ &= \frac{1}{\sqrt{2\pi}\sigma_{\rm sub}} \int_{\delta_c}^{\infty} \exp\left\{\frac{1}{2}\frac{(\delta_{M_1} - \delta_c)^2}{\sigma_{\rm sub}^2}\right\} d\delta_{M_1} \\ &= \frac{1}{2}, \qquad (11) \\ \sigma_{\rm sub}^2 &= \sigma^2(M_1) - \sigma^2(M_2) \qquad (12) \end{aligned}$$

where we use the sharp k-space filter (see YNG). Thus we can obtain the PS formula, naturally including the factor of 2 as can be seen from eqs.(10) and (11).

However, it is insufficient for more realistic estimation of the mass function to use eq.(11) because it is necessary to consider the spatial correlation of the density fluctuations due to the finite size of the objects. Therefore, we must consider the probability $P(r, M_1, M_2)$ of finding $\delta_{M_1} \geq \delta_c$ at a distance r from the centre of an isolated object of mass scale M_2. Then we can get the probability $P(M_1, M_2)$ by spatially averaging $P(r, M_1, M_2)$.

Because we believe that the isolated collapsed objects are formed around density peaks, the constraints to obtain the above probability $P(r, M_1, M_2)$ are given as follows:

(i) The linear density contrast, δ_{M_2}, of the larger mass scale M_2, should be equal to $\delta_c = 1.69$ at the centre of the object.

(ii) Objects of the mass scale M_2 must contain a maximum peak of the density field, *i.e.*, the first derivative of the density contrast $\nabla\delta_{M_2}$ must be equal to 0 and each diagonal component of the diagonalized Hessian matrix ζ of the second derivatives must be less than 0 at the centre of the object.

(iii) The density contrast of the smaller mass scale $M_1 (\leq M_2)$ which collapsed and is included in an object of mass scale M_2 must satisfy the condition $\delta_{M_1} \geq \delta_c$ at distance r from the centre of the larger object.

The probability which we get from the above conditions is:

$$\begin{aligned} P(r, M_1, M_2|\text{peak}) &= P(\delta_{M_1} \geq \delta_c | \delta_{M_2}, \text{peak}) \\ &= \sqrt{\frac{1-\gamma^2}{2\pi(1-\epsilon^2-\mu^2-\gamma^2+2\epsilon\mu\gamma)}} \\ &\quad \times \frac{\int_0^\infty dx\ f(x) \int_{\nu_{1c}}^\infty d\nu_1 \exp(-\frac{Q_a+Q_b}{2})}{\int_0^\infty dx\ f(x) \exp(-\frac{Q_b}{2})} \qquad (13) \end{aligned}$$

Since the detailed derivation of the above probability and explanations of the notation are complicated, we omit them here (see YNG). By spatially averaging eq.(13) in the region of M_2, we obtain $P(M_1, M_2)$ as follows:

$$P(M_1, M_2) = \frac{\int_0^{R_2} P(r, M_1, M_2|\text{peak}) 4\pi r^2 dr}{\int_0^{R_2} 4\pi r^2 dr}, \qquad (14)$$

where R_2 means the radius of the region M_2, $R_2 = (3M_2/4\pi\bar{\rho})^{(1/3)}$. The cumulative multiplicity functions, which will be defined in the next section, are estimated by using eq.(13) and are shown by the short-dashed lines in Fig.3 in the cases of the power spectrum $P(k) \propto k^n$ with $n = 0$ and -2, respectively. We call this formula the YNG formula hereafter.

4 RESULTS

4.1 Cumulative multiplicity functions

We calculate the cumulative multiplicity function $P(\geq M)$, which is defined as the mass fraction of objects whose mass is larger than a mass M to the total mass (in a fair sample of the Universe), by following the various methods which are mentioned in §§2 and 3. The multiplicity function $P(M) d \ln M$ (which we define in a logarithmic interval of

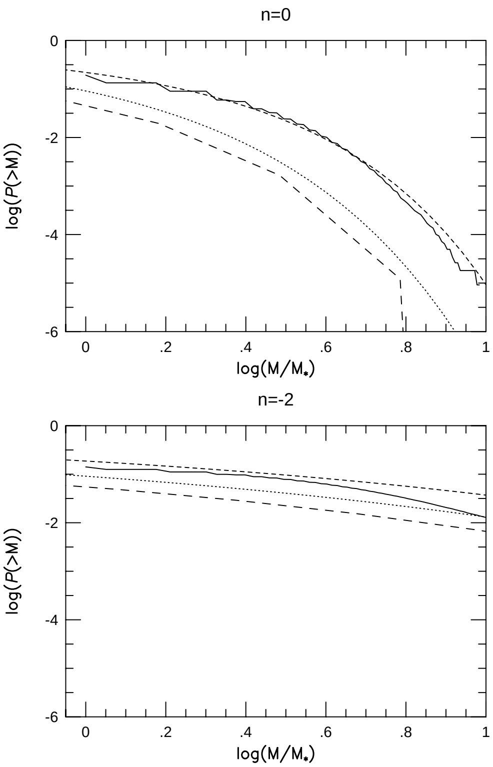

Figure 3. Cumulative multiplicity function: (a)spectral index n=0, (b)n=-2. The solid lines, the short-dashed lines, the dotted lines and the long-dashed lines show the cumulative multiplicity functions given by the MCM averaged over four realisations, the YNG formula, the PS formula and the Block model averaged over five realisations, respectively.

mass) and the cumulative multiplicity function are related to the mass function $n(M)dM$ as follows:

$$P(M)d\ln M \equiv \frac{d\rho}{\bar{\rho}} = \frac{M^2 n(M)}{\bar{\rho}} d\ln M, \tag{15}$$

$$P(\geq M) = \int_M^\infty P(M)\frac{dM}{M}. \tag{16}$$

We assume that the linear density fluctuations obey a random Gaussian distribution with a power-law power spectrum $P(k) \propto k^n$, where $n = 0$ and -2. We consider only the Einstein-de Sitter universe ($\Omega = 1, \Lambda = 0$) in this paper. In the MCM, we take the box size $L = 128$. M_* is defined as $\sigma(M_*) = 1$ (see eq.(3)) and here the mass in the block with the mass scale M_* is assigned to eight cells. In the Block model, the largest box size is assigned to the mass scales $M_0 = 10^5 M_*$ in the case of $n = 0$ and $10^7 M_*$ in the case of $n = -2$. We consider block sizes with $M_i = M_0/2^i (i = 1, 2, \cdots, N = 27)$.

In Figs.3(a) and (b), we show the cumulative multiplicity functions derived by the MCM, the YNG formula, the PS formula and the Block model for the case of $n = 0$ and $n = 2$, respectively.

The multiplicity functions of the MCM are shown only in the mass range ($0 \leq \log(M/M_*) \lesssim 1$) which corresponds to the range from eight cells to about 100 cells. The reason is as follows: On the smallest mass scale (one cell scale) in the numerical calculations, the power of the density fluctuations on the one cell scale cannot be correctly produced in agreement with the theoretically estimated power of the scale-free mass spectrum $\sigma(M) \propto M^{-(3+n)/6}$ due to the finiteness of the cell size in the numerical calculations. On the larger mass scales, larger haloes than about 100 cells have various shapes, rather than the spherical shape, so the assumption of spherically symmetric collapse fails. Hence, the cumulative multiplicity function given by different method on these scales cannot be directly computed without clarifying the identification of the isolated haloes. Moreover, since there are few haloes on the scales larger than about 100 cells in the numerical calculations, the error in the number of haloes increases. Therefore, we show here the cumulative multiplicity function only on the mass range from eight cells to about 100 cells ($0 \leq \log(M/M_*) \lesssim 1$).

It is found that the multiplicity functions given by the MCM are well fit by those given by the YNG formula. In the case of $n = 0$, the agreement is good while in the case of $n = -2$, there is a little deviation on the mass scale $\log(M/M_*) \gtrsim 0.8$ because of the numerical errors mentioned above. It is also found that the PS formula and the Block model are also in agreement with each other in the case of $n = -2$ rather than in the case of $n = 0$. Note that on the galaxy scales the spectral index n is nearly equal to -2 in the CDM model. Then in the CDM model the both functions are consistent with each other (Cole & Kaiser 1988). It must be noticed that the agreement of the Block model on the PS formula depends on the power spectrum. On the other hand, the function of the MCM deviates from those of the Block model and the PS formula in which the spatial correlations of the density fluctuations are not explicitly taken into account. From these results, we can conclude that the MCM naturally and correctly takes into consideration the spatial correlations and the deviation of the multiplicity function of the MCM from those of the Block model and the PS formula results from the effect of the spatial correlations of the density fluctuations. Furthermore, we will show in the next subsection that the multiplicity function given by the Block model can be reproduced by using the Jedamzik formula without consideration of the spatial correlations and thereby demonstrate that the difference between the multiplicity functions given by the Block model and the MCM results from the effect of the spatial correlations.

4.2 Block model

Here we analytically reproduce the multiplicity function given by the Block model by using the Jedamzik formula.

Since we consider the discrete mass of the blocks ($M_i = M_0/2^i$) in the Block model, the density contrast in the blocks which are identified as isolated collapsed haloes is generally greater than δ_c. We cannot recognise the just collapsed halo whose density contrast is precisely δ_c if we follow the procedure of the Block model. Therefore, the conditional probability $P(M_1, M_2)$ in this case must be approximately expressed as:

$$\begin{aligned} P(M_1, M_2) &= p(\delta_{M_1} \geq \delta_c | \delta_{M_2} \geq \delta_c) \\ &= \frac{p(\delta_{M_1} \geq \delta_c, \delta_{M_2} \geq \delta_c)}{p(\delta_{M_2} \geq \delta_c)} \end{aligned}$$

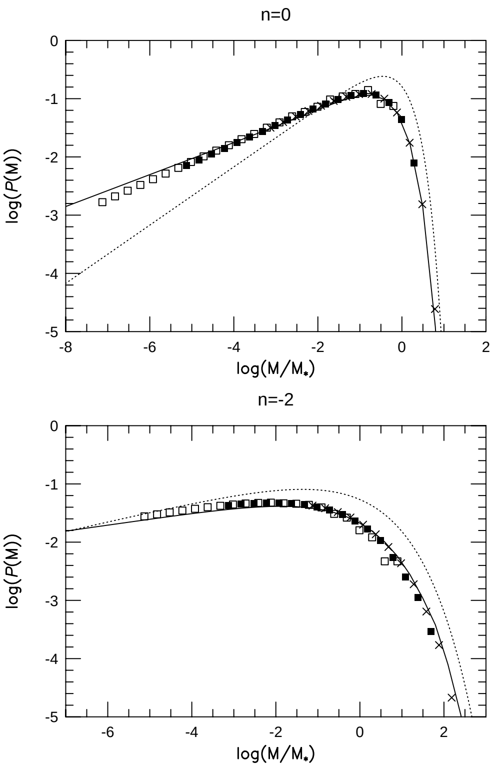

Figure 4. Block Model: (a)n=0, (b)n=-2. Crosses, solid squares and open squares show the multiplicity functions averaged over five realisations of the Block model with the largest box size $M_0 = 10^5 M_*, 10^3 M_*$ and $10 M_*$ in the case of $n = 0$ and $10^7 M_*, 10^5 M_*$ and $10^3 M_*$ in the case of $n = -2$, respectively. The solid lines and the dotted lines show the predictions by using the Jedamzik formula and the PS formula, respectively.

$$= \frac{\int_{\nu_{1c}}^{\infty} d\nu_1 \int_{\nu_{2c}}^{\infty} d\nu_2 \exp\left\{-\frac{(\nu_1-\epsilon'\nu_2)^2}{2(1-\epsilon'^2)} - \frac{\nu_2^2}{2}\right\}}{\sqrt{2\pi(1-\epsilon'^2)} \int_{\nu_{2c}}^{\infty} d\nu_2 \exp\left\{-\frac{\nu_2^2}{2}\right\}}. \quad (17)$$

It must be noticed here that we must consider the case of $\delta_{M_2} \geq \delta_c$ instead of the case of $\delta_{M_2} = \delta_c$ which appeared in eq.(11). In this case, of course, the spatial correlation is not taken into consideration. The detailed derivation of the above probability and notation are explained in Appendix A. Inserting the above conditional probability, eq.(17), into eq.(10), we can estimate the multiplicity function in this case.

In Fig.4, we show the multiplicity functions given by the above procedure (solid line) and the Block model (cross, solid square and open square). The cross, solid square and open marks in the Block model mean that the masses of the largest block M_0 equal $10^5 M_*, 10^3 M_*$ and $10 M_*$, respectively. Here in the Block model, the multiplicity function over a large dynamic range in mass can be produced by combining the functions derived by the Block model with $M_0 = 10^5 M_*, 10^3 M_*$ and $10 M_*$ in the case of $n = 0$ and $10^7 M_*, 10^5 M_*$ and $10^3 M_*$ in the case of $n = -2$ and block sizes with $M_i = M_0/2^i (i = 1, 2, \cdots 27)$ in each case. We find that the multiplicity functions given by the Block model and the analytical formula derived from the Jedamzik formula fit each other well. We believe that the difference between the cumulative functions given by the Block model and the MCM mainly results from the effect of the spatial correlations while the difference between the functions given by the Block model and the PS formula results from the difference in the identification of isolated collapsed objects.

4.3 Overlapping effect

In taking into consideration the finite size of the haloes, it appears necessary to consider the serious problem of the spatial overlapping of the dark haloes. For the overlapping criterion we must consider how we can identify the number and sizes of the haloes when some haloes overlap. In this subsection, we show the effect of the overlapping criterion (§2.2,(b)) on the multiplicity function. The overlapping criterion adopted in RT is that the investigating region can collapse when the overlapping region has a larger mass than half of the lesser of the masses of the halo and the investigating region. Here we quantify the overlapping criterion by defining a parameter, x, as the ratio of the mass of the overlapping region to the lesser of the masses of the halo and the investigating region. Then, RT's criterion corresponds to $x = 1/2$. By changing the value of x we show the effect of overlapping on the multiplicity function.

In Fig.5, we show the multiplicity functions on scales above eight cells in the cases that $x = 1/2$, $1/4$ and $1/8$. As the value of x decreases, the multiplicity function increases on the larger mass scales because it becomes easy for larger blocks to collapse.

We find that the multiplicity function on the larger mass scales changes according to the value of x. Note that this change is shown quantitatively in Fig. 5 only on scales smaller than about 100 cells because of the uncertainties due to the numerical errors on scales larger than 100 cells which are mentioned in §4.1. We consider the trend of the multiplicity function on large mass scales to increase with decreasing x on scales larger than 100 cells to also be significant.

So, the overlapping effect is a serious problem for constructing merger trees of dark haloes, particularly on mass scales larger than about 100 cells.

5 CONCLUSIONS & DISCUSSIONS

In this paper, we have shown that the multiplicity functions given by the MCM are consistent with those given by the YNG formula. However, the functions of the MCM do not fit those given by the PS formula and the Block model. This fact means that the MCM includes the information of the spatial correlations in the density fluctuation field, but the PS formula and the Block model do not include it. Thus we believe that the effects of the spatial correlations affect the merger trees as well as the mass functions and it is important to take into account the spatial correlations in the merger tree models.

Here it must be noted that the reason why the multiplicity functions given by the MCM are well fit by those given by the YNG formula in the case that the overlapping criterion has the value of $x = 1/2$ is as follows. When obtaining the multiplicity function from eqs.(13) and (14), we integrate the $P(r, M_1, M_2)$ with respect to r from 0 to R_2 in eq.(14).

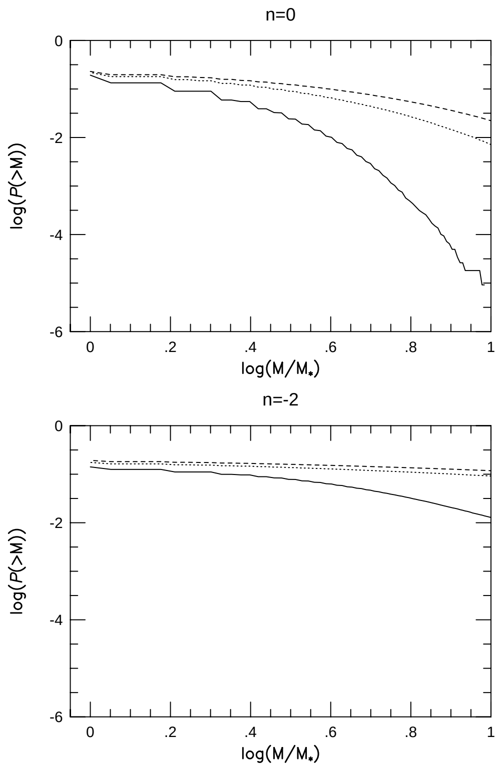

Figure 5. Effects of overlapping: (a)n=0, (b)n=-2. The solid lines, dotted lines and dashed lines show the multiplicity functions averaged over four realisations given by the MCM with the overlapping criterion defined by $x = 1/2, 1/4$ and $1/8$, respectively.

This corresponds to the case that $x = 1/2$. Of course, the adoption of the integration interval $[0, R_2]$ is, strictly speaking, only justified for spherically symmetric collapse. In fact, however, the effects of non-spherical collapse or tidal interaction among haloes may be important. We hypothesise that changing the value of x might be enough to account for these effects.

We also analytically produce multiplicity functions which fit those given by the Block model, by using the Jedamzik formula. When we reproduce the multiplicity functions given by the Block model, the conditional probability eq.(17) is adopted instead of eq.(11) in the PS formula. The difference from the PS formalism is that the density contrast of the isolated object is not only equal to δ_c but also greater than δ_c. This difference results from the discreteness of block mass in the Block model. It must be noticed that the conditional probability eq.(17) does not include the effect of the spatial correlations of the density fluctuations. We find that the multiplicity functions reproduced in this way are consistent with those given by the Block model. This result shows that the Block model does not include the spatial correlations of the density fluctuations and so the multiplicity function given by the Block model is different from those given by the MCM.

Moreover, we have shown that when the overlapping criterion is changed, the multiplicity function is affected, especially on larger mass scales. As the value of the overlapping parameter, x (defined as the ratio of the mass of the overlapping region to the lesser of the masses of the halo and the investigating region) decreases, the number density of high mass haloes increases because it becomes easy for larger blocks to collapse.

We believe that the MCM is a powerful tool for constructing merger trees of dark haloes because it includes the information of spatial correlations naturally and correctly as shown in this paper. In the MCM, the number of high mass haloes increases, relative to the results of previous works, *e.g.*, Kauffman et al. (1993) and Cole et al. (1994), so the number of giant, red elliptical galaxies may increase. In the MCM, however, a serious problem appears, *i.e.*, it is found that in taking into consideration the overlapping of dark haloes of non-zero size, the multiplicity function is sensitive to the choice of the overlapping criterion. Therefore, we must consider a more realistic overlapping criterion for constructing merger trees. We do not know *a priori* the value of the overlapping parameter. Furthermore the value of the overlapping parameter may not be constant as time passes. It might be a function of halo mass, of separation between a halo and a block, or some other parameter. It is a very difficult problem to identify the mass and size of the collapsed halo when several haloes are overlapped. However, it is very important to precisely identify the collapsed haloes for constructing the merger trees of haloes with good accuracy. As mentioned in §1, the merger tree of the dark haloes greatly affects the various important processes for the formation and evolution of galaxies, such as the thermal history of gases, merging between galaxies and so on. Hence, it is necessary and important to understand how the overlapping criterion influences these processes. Furthermore, we assume spherically collapse of the dark haloes. But, in fact, there is the possibility that the collapsed objects have filament or sheet structures. We must also deal with these cases in order to correctly estimate the merger trees. We are now investigating the merger tree models in which the effect of non-spherical collapse is included and the identification of the collapsed haloes is well defined.

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APPENDIX A: MASS FUNCTION OF BLOCK MODEL

As mentioned in §3, we show the analytic formula which reproduces the mass functions given by the Block model.

The analytic formula is approximately expressed as follows. In YNG and PS, it is assumed that the density contrast δ_{M_2} of the isolated object which has collapsed with mass M_2 is just $\delta_c = 1.69$ (see eq.(13)). In the Block model, however, this condition should be changed to $\delta_{M_2} \geq \delta_c$ because in general we can not see the block which has just collapsed and its density contrast is just δ_c due to the discreteness of the mass scales of the blocks with $M_{i+1} = M_i/2$ in the Block model.

So the probability should be changed as follows:

$$\begin{aligned} P(M_1, M_2) &= p(\delta_{M_1} \geq \delta_c | \delta_{M_2} \geq \delta_c) \\ &= \frac{p(\delta_{M_1} \geq \delta_c, \delta_{M_2} \geq \delta_c)}{p(\delta_{M_2} \geq \delta_c)}. \end{aligned} \tag{A1}$$

In order to estimate the probability on the right hand side of eq.(A1), we have to consider the two-variables Gaussian distribution function. The probability of m-variables Gaussian is generally (see BBKS)

$$p(\mathbf{V})d\mathbf{V} = \frac{\exp(-Q/2)}{\sqrt{(2\pi)^m \det(\mathbf{M})}} d\mathbf{V}, \tag{A2}$$

where

$$Q = \mathbf{V}\mathbf{M}^{-1}\mathbf{V}^T, \tag{A3}$$

$$\mathbf{V} = (y_1, y_2, \ldots, y_m), \tag{A4}$$

$$M_{ij} = \langle (y_i - \langle y_i \rangle)(y_j - \langle y_j \rangle) \rangle \tag{A5}$$

and $y_i (i = 1, 2, \ldots, m)$ is the Gaussian random variables. We use the angle brackets $\langle\rangle$ as the ensemble average of the universe, but in practice, assuming homogeneity and ergodicity in space, we can take it as the spatial average. So $\langle y_i \rangle$ corresponds to the spatial average of y_i, and M_{ij} is covariance between y_i and y_j. In this case, we may consider the two variables, δ_{M_1} and δ_{M_2}, so the covariance matrix $\mathbf{M}$ is written as follows:

$$\mathbf{M} = \begin{pmatrix} \langle \delta_{M_1}^2 \rangle & \langle \delta_{M_1}\delta_{M_2} \rangle \\ \langle \delta_{M_1}\delta_{M_2} \rangle & \langle \delta_{M_1}^2 \rangle \end{pmatrix}. \tag{A6}$$

Note that $\langle \delta_{M_1}^2 \rangle$ and $\langle \delta_{M_1}^2 \rangle$ are variances, $\sigma^2(M_1)$ and $\sigma^2(M_2)$, respectively, and $\langle \delta_{M_1}\delta_{M_2} \rangle$ is a cross correlation at the same point.

Here, we normalize the density contrast and the cross correlation,

$$\nu_i \equiv \frac{\delta_{M_i}}{\sigma(M_i)}, \quad \epsilon \equiv \frac{\sigma_h^2}{\sigma(M_1)\sigma(M_2)}, \quad \sigma_h^2 \equiv \langle \delta_{M_1}\delta_{M_2} \rangle. \tag{A7}$$

Using the above notation, we obtain the two variables Gaussian distribution function,

$$p(\nu_1, \nu_2) d\nu_1 d\nu_2 = \frac{1}{2\pi\sqrt{1-\epsilon^2}} \exp\left(-\frac{(\nu_1 - \epsilon\nu_2)^2}{2(1-\epsilon^2)} - \frac{\nu_2^2}{2} \right) d\nu_1 d\nu_2. \tag{A8}$$

So we obtain

$$\begin{aligned} p(\delta_{M_1} \geq \delta_c, \delta_{M_2} \geq \delta_c) &= p(\nu_1 \geq \nu_{1c}, \nu_2 \geq \nu_{2c}) \\ &= \frac{1}{2\pi\sqrt{1-\epsilon^2}} \\ &\quad \Upsilon \times \int_{\nu_{1c}}^{\infty} \int_{\nu_{2c}}^{\infty} \exp\left(-\frac{(\nu_1 - \epsilon\nu_2)^2}{2(1-\epsilon^2)} - \frac{\nu_2^2}{2} \right) d\nu_1 d\nu_2 \end{aligned} \tag{A9}$$

where ν_{ic} is $\delta_c/\sigma(M_i)$. Furthermore, dividing the above equation by the integration of one variable Gaussian distribution,

$$\begin{aligned} p(\delta_{M_2} \geq \delta_c) &= p(\nu_2 \geq \nu_{2c}) \\ &= \frac{1}{\sqrt{2\pi}} \int_{\nu_{2c}}^{\infty} \exp\left(-\frac{\nu_2^2}{2} \right) d\nu_2, \end{aligned} \tag{A10}$$

we obtain the $P(M_1, M_2)$.

Next, we estimate the normalized cross correlation function ϵ. In eqs.(3) and (5), the variance σ_i with the mass M_i is given by

$$\sigma_i^2 = \frac{4\pi V}{(2\pi)^3} \int_0^{\infty} W_i^2(k) P(k) k^2 dk, \tag{A11}$$

and the variance of the additional density contrast Σ is given by

$$\Sigma_i^2 = \sigma_i^2 - \sigma_{i-1}^2, \quad i \geq 1. \tag{A12}$$

Because we give the additional density contrast independently of another density contrast with the other mass scale, the variance of the additional density contrast must depend on the power spectrum only at the interval of the wavenumber $[k_{i+1}, k_i]$, corresponding to the mass scales $[M_i, M_{i+1}]$. So we should adopt the sharp-k space filter as the window function $W_i(k)$ defined by,

$$W_i(k) = \begin{cases} 1, & k \leq k_c(M_i) \\ 0, & k > k_c(M_i), \end{cases} \tag{A13}$$

In this case, the variance and the cross correlation are

$$\sigma_i^2 = \frac{4\pi V}{(2\pi)^3} \int_0^{k_i} P(k) k^2 dk, \tag{A14}$$

$$\begin{aligned}\sigma_h^2 &= \frac{4\pi V}{(2\pi)^3}\int_0^\infty W_1(k)W_2(k)P(k)k^2dk \\ &= \frac{4\pi V}{(2\pi)^3}\int_0^{k_2} P(k)k^2dk \\ &= \sigma_2^2 \end{aligned} \tag{A15}$$

where $i = 1, 2$ and $M_1 \leq M_2$, *i.e.*, $k_1 \geq k_2$. So the normalized cross correlation function ϵ is

$$\epsilon = \frac{\sigma_h^2}{\sigma_1\sigma_2} = \frac{\sigma_2}{\sigma_1}. \tag{A16}$$

Substituting the above eq.(A16) into eq.(A9), we obtain

$$p(\delta_{M_1} \geq \delta_c, \delta_{M_2} \geq \delta_c) = \frac{1}{2\pi\sigma_2\sigma_{\rm sub}} \times \int_{\delta_c}^\infty\int_{\delta_c}^\infty \exp\left(-\frac{(\delta_1-\delta_2)^2}{2\sigma_{\rm sub}^2} - \frac{\delta_2^2}{2\sigma_2}\right) d\delta_1 d\delta_2, \tag{A17}$$

where

$$\sigma_{\rm sub}^2 = \sigma_1^2 - \sigma_2^2. \tag{A18}$$

Therefore the conditional probability $P(M_1, M_2)$ is

$$P(M_1, M_2) = N^{-1}\frac{1}{2\pi\sigma_2\sigma_{\rm sub}} \times \int_{\delta_c}^\infty\int_{\delta_c}^\infty \exp\left(-\frac{(\delta_1-\delta_2)^2}{2\sigma_{\rm sub}^2} - \frac{\delta_2^2}{2\sigma_2}\right) d\delta_1 d\delta_2, \tag{A19}$$

where

$$N = \frac{1}{\sqrt{2\pi}\sigma_2}\int_{\delta_c}^\infty \exp\left(-\frac{\delta_2^2}{2\sigma_2}\right) d\delta_2. \tag{A20}$$

Note that this probability is the same as eq.(8) in Jedamzik (1995).

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